Spatial Projection Rectification for Densifying Ground Control Points

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Abstract. The ground control points of marine and coastal remote sensing images could not be properly selected because of the special characteristics of the ocean, thus the accuracy of geometric rectification was restricted. A new geometric exact rectification method of spatial projection was put forward, and it could gain dense ground control points. The rectification principle and steps of this method were systematically studied, and the rectification model was researched. It showed that the accuracy of spatial projection rectification model was 1.078 pixels from the experiment result. So it is an effective method to those remote sensing images lack of ground control points.

Keywords: geometric exact rectification, spatial projection, rectification accuracy, ground control point, coastal zone, remote sensing image

1. Introduction

Remote sensing images will not be used only by radiation and geometric glancing correction until they are further rectified by geometric exact rectification. The current commonly used geometric exact rectification models include the polynomial method, collinear equation method, and rational faction method, direct linear transformation method and delauney triangle method, etc. Relatively high location accuracy of these models could be reached, but a certain amount of ground control points (GCPs) must be required in the range of remote sensing image. Furthermore, the process of location is based on ground instead of space, and the rectification process is based on static mode instead of dynamic mode. That is, the location of spatial remote sensing image will be decided by the geographic coordinates of GCPs [1].

In fact, China has the 18 000km long coastal line of the main land, and the 14 000km of the islands. And there are plenty of marine and coastal zone remote sensing images that are lack of GCPs [2][3]. These images always consist of sea area, coastal area and land area. In the sea area, GCPs could not be selected except for the islands [4]. Because of the wide range and frequent movement of alluvion in the coastal area, it is difficult to find the stable, reliable and obvious ground symbols. GCPs are always gathered in the land area, which will result in the low accuracy of geometric exact rectification [5]. In lots of these images of inconsistent integral distortion for lacking or nonuniform distributing of GCPs, the rectification accuracy can’t meet the high-precision demands.

In the paper, based on dynamic spatial projection principles, a new method of remote sensing image geometric exact rectification will be studied and the accuracy will be about 1 pixel. It is hoped to be the effective method to those remote sensing images lack of GCPs.

2. Basic theory and mathematics principle of gaining dense GCPs

2.1. Impacts of GCPs on rectification accuracy

The accuracy of geometric exact rectification of remote sensing image is determined by many factors, e.g.
the selection of GCPs, the configuration and distribution of GCPs and their reliabilities, the topography and relief characteristics of the region, and the degree of complexity of geometric rectification model. Among them, the most important factors are the accuracy, the number and the distribution of GCPs [6]. And it will be studied respectively as the following.

1. The Accuracy of GCPs: GCPs for geometric exact rectification of remote sensing image are always selected and measured in the high-precision and up-to-date topographic map. And the map scale should be larger than or equal to the scale transformed from image accuracy. The exactly located special points include the clear intersections between two roads or between road and river and the prominent ground feature. And they should be selected both in the image and the map (or the digital topographic map). If chosen, the image should use the combination of several wave bands of tiny discrimination of strong ground feature. As to the fuzzy image where it is difficult to select the GCPs, the above rules are of great importance [7].

2. The Number of GCPs: In this paper, the number of GCPs is determined by the degree of the rectification polynomials, and also related to the required accuracy and the rectification area. The number of GCPs L should meet this inequation $L \geq \frac{(n+1)(n+2)}{2}$, there into n expresses the degree of rectification polynomials. The quadratic and cubic rectification polynomials are always selected and may meet the accuracy requirement. When $L > \frac{(n+1)(n+2)}{2}$, the coefficient of polynomials will be defined by the redundant GCPs using least square principle. The surface reflects a trend of fitting and does not get across every GCP, so errors of one or two GCPs will result in the rectification error of the whole image. When $L = \frac{(n+1)(n+2)}{2}$, the fitting surface will strictly get across all the GCPs, if a certain GCP has large error, the rectification accuracy of the whole image will be relatively influenced. So sometimes adding GCPs would improve the rectification accuracy, but too many GCPs will not definitely result in the high accuracy and it may add workload in selecting GCPs which will get even harder to be selected [8].

3. The Distribution of GCPs: Distribution of GCPs should be well-proportioned; otherwise those areas with dense GCPs will reach high rectification accuracy, but with rare GCPs will get large fitting errors. Further, different resampling method of pixel brightness will bring different brightness distortion, geometric location error and different computer processing speed. For example, the greyscale information of previous image and the fastest computer speed could be remained by least neighbourhood interpolation method, yet the error will be as large as ±0.5 pixel. Sometimes bi-linear interpolation method will be adopted, but if the distortion is very big compared with the previous image, then the image of middle and high latitude zone should be rectified to the geographic coordinates system. For this occasion, the cubic convolution method of resampling should be adopted so as to keep the least distortion of brightness [9].

2.2. Mathematics principle of gaining dense GCPs

The scanning imagery of satellite is a dynamic processing including four relative movements: the swing of scanning lens, satellite movement along orbit, earth rotation and orbit precession, which will inevitably produce distortions. Thus a dynamic processing method should be set up for the dynamic acquirement of remote sensing image. The mathematics principle of dynamic spatial projection about gaining dense GCPs is researched as the following.

Traditional map projection is called static projection for there isn’t relative movement between the projection plane and the earth plane. While spatial projection is a dynamic map projection and belongs to four-dimensional map projection and t (time) is its main parameter. The effect of satellite motion, the earth’s rotation and orbital precession are taken into consideration. So the physical processes of satellite imaging can be well stimulated, and the geometric relationship can be accurately established between pixel elements and ground control points. Spatial projection model has an advantage that it has matched with the numerical fitting method now applied in image rectification. And there is another advantage that the spatial projection system can force the strip images in the same flight orbit to be in a unified coordinate system. Hence only a small number of ground control points are needed for building the projection system.

There are many different kinds of spatial projections fitting for imageries of various dynamic sensors. Spatial Oblique Mercator (SOM) projection is one of the spatial projections related to the experiment in this paper. SOM projection is a newly designed projection for the sequence images taking by scanning equipment of the orbit spaceflight of U.S. Geological Survey [10]. At the same time when the point to point ground scanning vertical to the flight direction has been accomplished by the multi-spectrum scanning (MSS), the
scanning parallel to the flight direction has been completed. As the figure shows, one swing of the lens will have simultaneously six scanning lines across the ground, form a 474 meters long and 185 km wide land cover.

A dynamic column is supposed instantaneously tangent to the trace line of the satellite substellar point. A directly relationship will be established between the scanning image and the cylindrical surface, so the dynamic cylindrical surface will be tangent to the earth surface along the satellite trace, and will fluctuate with a compensation speed along the axis, and its speed varies with latitude. The physical process of satellite imagery is perfectly simulated by this model, and the distortion influence is well eliminated caused by the satellite flying, orbit motion, earth rotation and earth curvature. Direct relationship between image and projection system is established according to a few GCPs. Still the ground traces are described by this model of different sheets along the same orbit with sequent inerrant scales.

Fig. 1: MSS scanning sketch map

Fig. 2: Geometric model of SOM
2.3. Mathematics model of SOM

As satellite imagery is along the motion of satellite and the rotation of earth, time will be the main parameter,

\[
\begin{align*}
  x &= f_1(\phi, \lambda, t) = f_1(x', y', z', t) \\
  y &= f_2(\phi, \lambda, t) = f_2(x', y', z', t)
\end{align*}
\]

(1)

and then come the functions:

Project the scanning zone to the cylindrical surface and then spread out to a plane, then the function of SOM projection is\(^{[11]}\):

\[
X = a \int_0^2 \frac{HJ - S^2}{\sqrt{J^2 + S^2}} d\lambda - \frac{S}{F \sqrt{J^2 + S^2}} \ln \tan(\frac{\pi}{4} + \frac{\phi}{2})
\]

\[
Y = a \int_0^2 \frac{S(H + J)}{\sqrt{J^2 + S^2}} d\lambda + \frac{J}{F \sqrt{J^2 + S^2}} \ln \tan(\frac{\pi}{4} + \frac{\phi}{2})
\]

(2)

The linearization of formula (2) is:

\[
\begin{align*}
  X / a &= A_0 \lambda'' / 2 + A_2 \sin 2\lambda'' / 2 + A_4 \sin 4\lambda'' / 4 + A_6 \sin 6\lambda'' / 6 - \\
  &\quad (B_1 \cos \lambda'' + B_3 \cos 3\lambda'' + B_5 \cos 5\lambda'') \ln \tan(\pi / 4 + \phi'' / 2) \\
  Y / a &= C_1 \sin \lambda'' - C_3 \sin 3\lambda'' / 3 + C_5 \sin 5\lambda'' / 5 + C_7 \sin 7\lambda'' / 7 + \\
  &\quad (D_1 / 2 + D_3 \cos 2\lambda'' + D_5 \cos 4\lambda'') \ln \tan(\pi / 4 + \phi'' / 2)
\end{align*}
\]

(3)

The inverse transformation formula of (2) and (3) is:

\[
\begin{align*}
  X / a &= (X / a) + (Y / a)(G_1 \cos \lambda'' + G_3 \cos 3\lambda'' + G_5 \cos 5\lambda'') + \ldots \\
  E_3 \sin 2\lambda'' - E_4 \sin 4\lambda'' - E_6 \sin 6\lambda'' - \ldots \\
  \ln \tan(\pi / 4 + \phi'' / 2) &= (Y / a - C_1 \sin \lambda'' - C_3 \sin 3\lambda'' / 3 - C_5 \sin 5\lambda'' / 5) \\
  &\quad / (D_1 / 2 + D_3 \cos 2\lambda'' + D_5 \cos 4\lambda'')
\end{align*}
\]

(4)

There into:

\[
A_n = \frac{2}{\pi} \int_0^{2\pi} \frac{HJ - S^2}{\sqrt{J^2 + S^2}} \cos n\lambda'' d\lambda'' \quad J = (1 - e^2)^3
\]

\[
B_n = \frac{1}{\pi m} \int_0^{2\pi} \frac{S}{\sqrt{J^2 + S^2}} \cos n\lambda'' d\lambda'' \quad Q = e^2 \sin^2 i / (1 - e^2)
\]

\[
C_n = \frac{1}{\pi} \int_0^{2\pi} \frac{S(H + J)}{\sqrt{J^2 + S^2}} \cos n\lambda'' d\lambda'' \quad T = e^2 \sin^2 (2 - e^2) / (1 - e^2)^2
\]

\[
D_n = \frac{1}{\pi} \int_0^{2\pi} \frac{J}{\sqrt{J^2 + S^2}} \cos n\lambda'' d\lambda'' \quad u = e^2 \cos^2 i / (1 - e^2)^2
\]

\[
G_n = \frac{1}{J \pi m} \int_0^{2\pi} S \cos n\lambda'' d\lambda'' \quad W = \left[ (1 - e^2 \cos^2 i) / (1 - e^2) \right]^2 - 1
\]

\[
E_2 = A_2 + (C_1 G_1 + C_3 G_3 - C_1 G_3 + C_5 G_3 - C_3 G_5) / 2
\]

\[
E_4 = A_4 + (C_1 G_1 + C_3 G_3 + C_5 G_1 - C_1 D_3) / 2
\]

\[
E_6 = A_6 + (C_5 G_1 + C_1 G_5 + C_3 G_3) / 2
\]

\[
S = \frac{P_2}{P_1} \sqrt{1 + T \sin^2 \lambda'' / \left( (1 + W \sin^2 \lambda'')(1 + Q \sin^2 \lambda'') \right)} \sin i \cos \lambda''
\]

\[
H = \sqrt{1 + Q \sin^2 \lambda'' / \left( (1 + W \sin^2 \lambda'')(1 + Q \sin^2 \lambda'') \right) - \frac{P_2}{P_1} \cos i}
\]
There into, \( \lambda^* \) is transformation longitude of spatial projection, \( \phi^* \) is the transformation latitude, \( \alpha \) is the semi-major radius of earth ellipsoid, \( P_1 \) is the rotation period of the earth, \( P_2 \) is the circulation period of the satellite, \( e \) is the first eccentricity of the earth, \( i \) is the plane obliquity of satellite orbit.

3. The realization of gaining dense GCPs

3.1. Process of gaining dense GCPs

SOM projection designed by U.S. Geological Survey is just for Landsat TM image, so the geometric attitude has been well reflected. The geometric exact rectification using SOM projection method will be well carried out, and the following is the rectification model\(^{(12)}\).

First, the relation between image coordinates and SOM coordinates is established using quadratic polynomial and the function is as the following:

\[
\begin{align*}
X &= a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 x y + a_5 y^2 \\
Y &= b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 x y + b_5 y^2
\end{align*}
\]  

(6)

There into, \( X, Y \) are coordinates of SOM, \( x, y \) are coordinates of pixels.

Second, determine every SOM coordinates using function (6) and inverse resolve each geographic coordinates using the inverse function of SOM function (4).

Third, define the output range of image based on the coordinates of the inverse function, and then rectify every pixel.

The flow of rectification is showed in Figure 3 using SOM projection model.

Put the corresponding coordinates of GCPs into formula (6) and then the transformation coefficients are resolved, thus the relation between images coordinates and SOM coordinates is defined. According to the inverse formula of SOM projection (4), dense GCPs are gained; finally geometric exact rectification is carried out using these dense GCPs.

3.2. Accuracy test model

Test of the rectification accuracy is firstly to calculate the Gauss coordinates \( (x'_g, y'_g) \) of the rectified image, secondly to request the difference between \( (x'_g, y'_g) \) and former Gauss coordinate \( (x_g, y_g) \), and finally to gain the error or the accuracy. The mathematics model is:

\[
\begin{align*}
D_{x_g} &= x'_g - x_g, \\
D_{y_g} &= y'_g - y_g, \\
D_s &= \sqrt{(D_{x_g})^2 + (D_{y_g})^2} \\
D &= \sqrt{\sum_{i=0}^{4}(D_{s_i})^2} / n
\end{align*}
\]  

(7)

Fig. 3: Geometric rectification of SOM projection
There into, $D_{x,g}$, $D_{y,g}$ is the error of $x$ direction and $y$ direction, $D_s$ is the positional distance accuracy of points, $D$ is the mean error of the image, n is the number of test points.

4. Result analysis of rectification experiment

The data for rectification experiment is from Landsat TM image of Xiamen coastal zone, Figure 4 below is the miniature of Xiamen TM image. The land cover of the image is a coastal zone. Not only the distribution of sea and land is not even but the GCPs are not evenly distributed. The resolution of the image is 30 meters. The size of the image is about 4818 pixels $\times$ 4818 rows. GCPs were selected from the topographic map scale of 1:10000. About 12 GCPs are selected as reference points given in Table 1. Among these points, the second, fourth, seventh, ninth and twelfth points are check points.

4.1. Accuracy analysis before gaining dense GCPs

The ground control points and test points were all selected from the terrain map. The basic data and experimental result were as the following:

Table 1 Comparing list of geographical coordinates and pixel coordinates of reference points

<table>
<thead>
<tr>
<th>series</th>
<th>pixels</th>
<th>rows</th>
<th>longitude</th>
<th>latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>255.1</td>
<td>225.0</td>
<td>118.027917</td>
<td>24.552083</td>
</tr>
<tr>
<td>2</td>
<td>382.8</td>
<td>313.4</td>
<td>118.061056</td>
<td>24.523111</td>
</tr>
<tr>
<td>3</td>
<td>665.0</td>
<td>143.5</td>
<td>118.151556</td>
<td>24.557222</td>
</tr>
<tr>
<td>4</td>
<td>387.4</td>
<td>251.5</td>
<td>118.065444</td>
<td>24.539777</td>
</tr>
<tr>
<td>5</td>
<td>685.5</td>
<td>376.5</td>
<td>118.149251</td>
<td>24.492917</td>
</tr>
<tr>
<td>6</td>
<td>404.3</td>
<td>403.8</td>
<td>118.063278</td>
<td>24.498611</td>
</tr>
<tr>
<td>7</td>
<td>367.8</td>
<td>533.4</td>
<td>118.046722</td>
<td>24.465083</td>
</tr>
<tr>
<td>8</td>
<td>148.6</td>
<td>392.7</td>
<td>117.989250</td>
<td>24.512847</td>
</tr>
<tr>
<td>9</td>
<td>240.3</td>
<td>761.7</td>
<td>117.998944</td>
<td>24.409224</td>
</tr>
<tr>
<td>10</td>
<td>265.8</td>
<td>669.5</td>
<td>118.010778</td>
<td>24.432833</td>
</tr>
<tr>
<td>11</td>
<td>413.5</td>
<td>564.5</td>
<td>118.058361</td>
<td>24.454778</td>
</tr>
<tr>
<td>12</td>
<td>460.5</td>
<td>596.5</td>
<td>118.071167</td>
<td>24.443972</td>
</tr>
</tbody>
</table>

The accuracy analysis of polynomial rectification method is showed in table 2:

Table 2: The absolute value of rectification error using polynomial method (Unit: m)

<table>
<thead>
<tr>
<th>Degree of polynomial model</th>
<th>X Maximum</th>
<th>Mean square error</th>
<th>Y Maximum</th>
<th>Mean square error</th>
<th>Plane Maximum</th>
<th>Mean square error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic polynomial</td>
<td>28.803457</td>
<td>18.168973</td>
<td>93.435683</td>
<td>47.457865</td>
<td>97.589321</td>
<td>51.456341</td>
</tr>
</tbody>
</table>

After analyzing the values in Table 2, the absolute value of maximum error in $x_g$ direction was 28.80$m$, in $y_g$ direction was 98.23$m$. The mean square error with this method was 51.45$m$, corresponding to 1.715 pixels. The rectified image with polynomial method was showed as Figure 5.
4.2. Accuracy analysis after gaining dense GCPs

According to formula (6), the relation of SOM projection coordinates and image coordinates was established using the seven GCPs. And with this relation and SOM inverse transformation formula, dense GCPs were acquired in the origin image. 33 GCPs were gained in the 4818 pixels × 4818 rows with the interpolation of 135 pixels × 135 rows. Then geometric exact rectification of the image was carried out using the 33 GCPs. The statistic analysis of accuracy with the five check points was in Table 3.

<table>
<thead>
<tr>
<th>series</th>
<th>Errors of X direction Dxg</th>
<th>Errors of Y direction Dyg</th>
<th>Errors of distance Ds</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5.962767</td>
<td>3.442579</td>
<td>6.885197</td>
</tr>
<tr>
<td>3</td>
<td>-19.822531</td>
<td>-29.068413</td>
<td>35.183879</td>
</tr>
<tr>
<td>4</td>
<td>24.127416</td>
<td>20.261594</td>
<td>31.506577</td>
</tr>
<tr>
<td>5</td>
<td>12.318513</td>
<td>-42.321628</td>
<td>44.077953</td>
</tr>
</tbody>
</table>

After analyzing the values in Table 3, the absolute value of maximum error in $x_g$ direction was 24.13 m, in $y_g$ direction was 42.32 m after gaining dense GCPs. The maximum error in distance was 44.08 m. The mean square error with this method was 32.36 m, corresponding to 1.078 pixels. The rectified image with SOM projection method was showed as Figure 6.

4.3. Comparison of rectification accuracies

From the above Table 2 and Table 3, in the rectification accuracy of polynomial method, the mean square error was 51.45 m, corresponding to 1.715 pixels. In the rectification accuracy of SOM projection, the mean square error was 32.36 m, corresponding to 1.078 pixels. So the accuracy of SOM projection is apparently higher than polynomial method. Moreover, the speed is quicker and the complexity is easier.
5. Conclusion

Aiming at remote sensing images lack of ground control points, a new geometric exact rectification method based on spatial projection was discussed in this paper. The rectification method of Spatial Oblique Mercator projection perfectly simulated the physical process of satellite imagery, and eliminated the distortion influence caused by the satellite flying, orbit motion, earth rotation and earth curvature. Direct relationship between image and projection system was established according to a few GCPs. This model can efficiently gain dense ground control points. The experimental results indicated that the accuracy of spatial projection method was about 1 pixel. Obviously, it was excelled the commonly used rectification methods. And this model was based on strictness theory and rapidness arithmetic and was also a rapid and efficient rectification model in the area lack of elevation information.

6. Acknowledgements

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7. References


