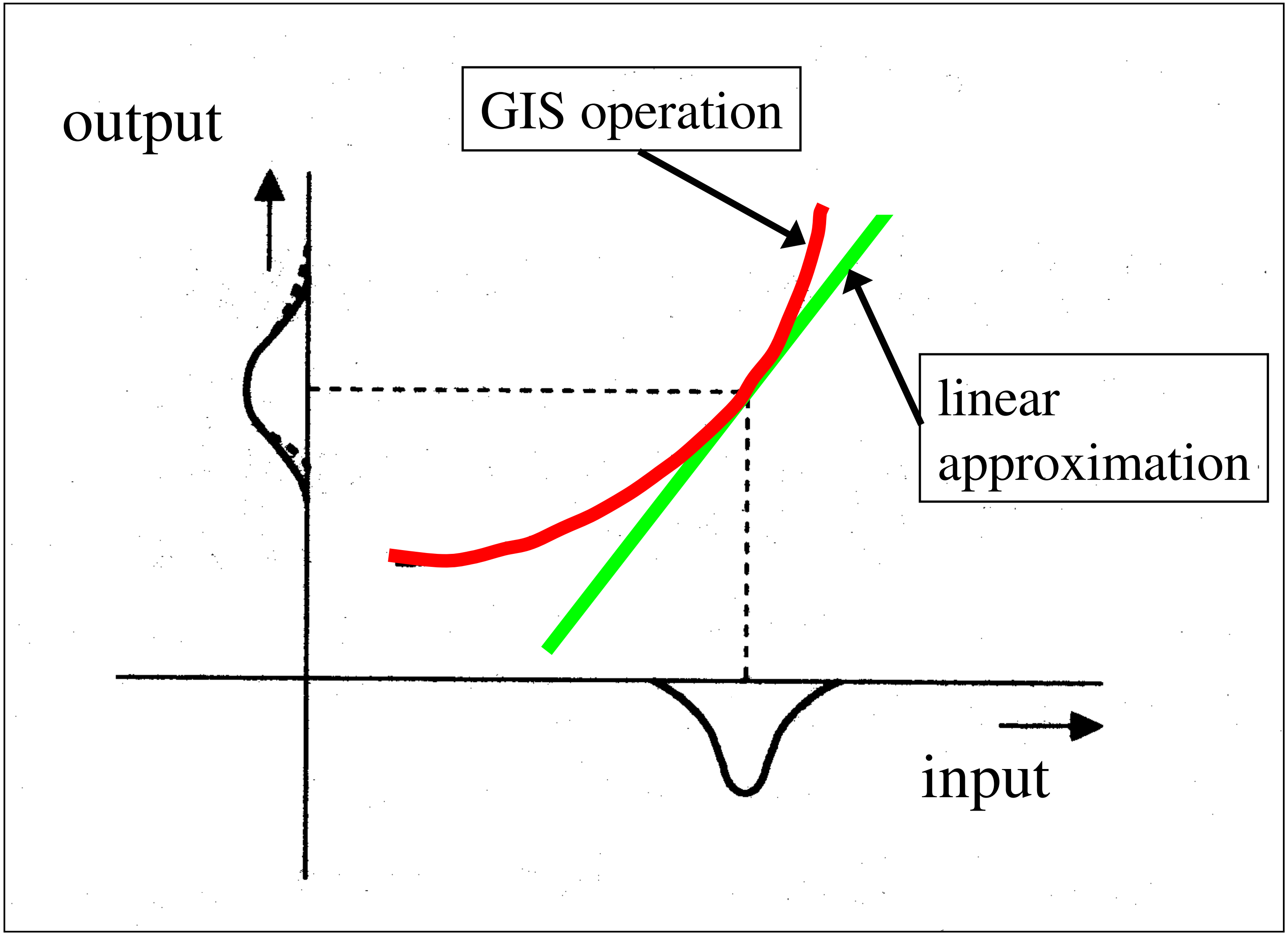
3. Actual error propagation analysis

We will discuss two methods:

- Taylor series approximation (this lecture)
- Monte Carlo method (lecture 4, tomorrow)

Taylor series method, one-dimensional case



Taylor series approximation

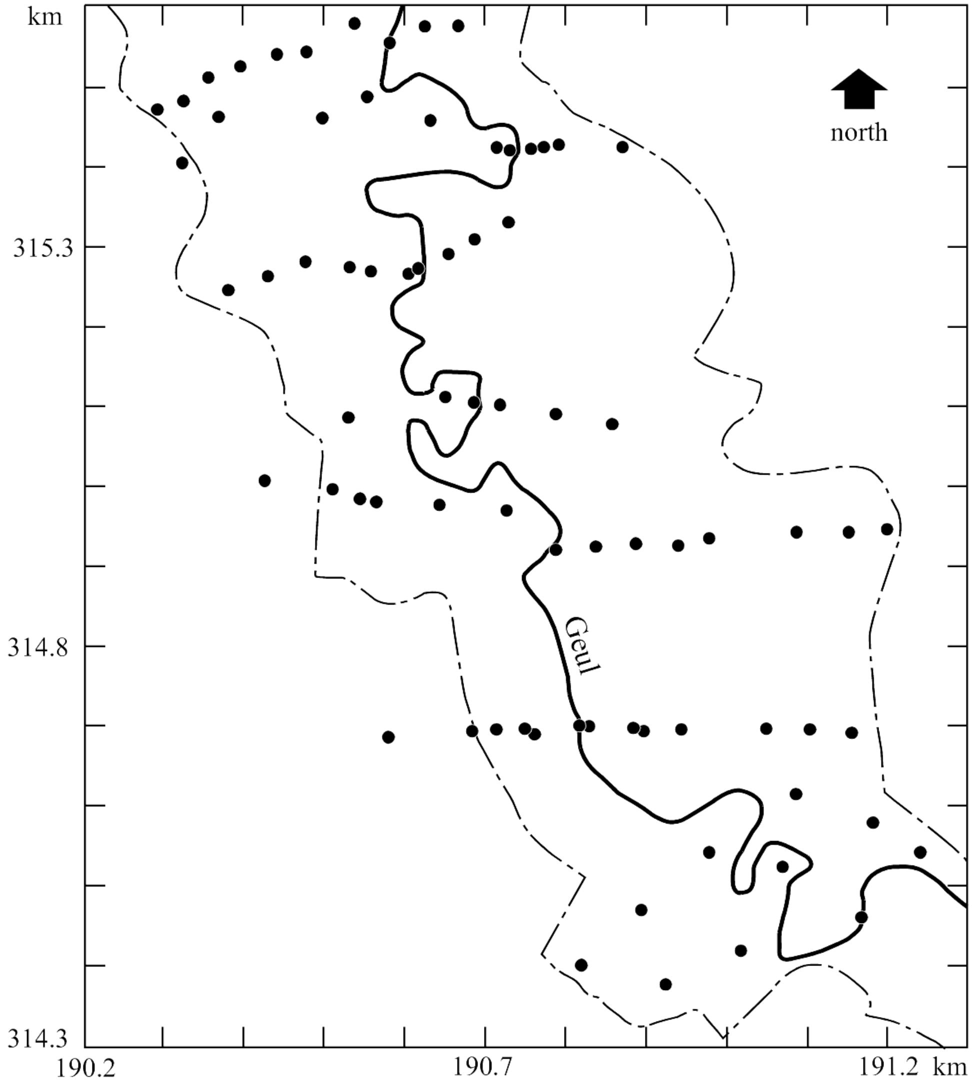
By linearising the GIS operation g we get (case of uncorrelated errors):

$$\mathrm{Var}(U) = \sum_{i=1}^{m} \mathrm{Var}(A_i) \cdot \left(\frac{\partial g}{\partial A_i} \right)^2$$

magnitude of input error matters

but also sensitivity of model to input

Geul lead pollution example



Playing children ingest lead

$$I = PB \cdot S$$

where:

I	= lead ingestion
PB	= lead concentration of soil
S	= soil consumption

How do errors in mapped lead concentration of soil and soil consumption propagate to lead ingestion?

Taylor series approximation

$$\mathrm{Var}(U) = \sum_{i=1}^{m} \mathrm{Var}(A_i) \cdot \left(\frac{\partial g}{\partial A_i} \right)^2$$

$$\mathrm{Var}(I) = \mathrm{Var}(Pb) * \left(\mathrm{mean}(S)\right)^2 + \mathrm{Var}(S) * \left(\mathrm{mean}(Pb)\right)^2$$

Analytical result:

$$\mathrm{Var}(I) = \mathrm{Var}(Pb) * \left(\mathrm{mean}(S)\right)^2 + \mathrm{Var}(S) * \left(\mathrm{mean}(Pb)\right)^2 + \\ + \mathrm{Var}(Pb) * \mathrm{Var}(S)$$

EXERCISE 2

Error source contributions

Recall from before:

$$\mathrm{Var}(U) = \sum_{i=1}^{m} \mathrm{Var}(A_i) \cdot \left(\frac{\partial g}{\partial A_i} \right)^2$$

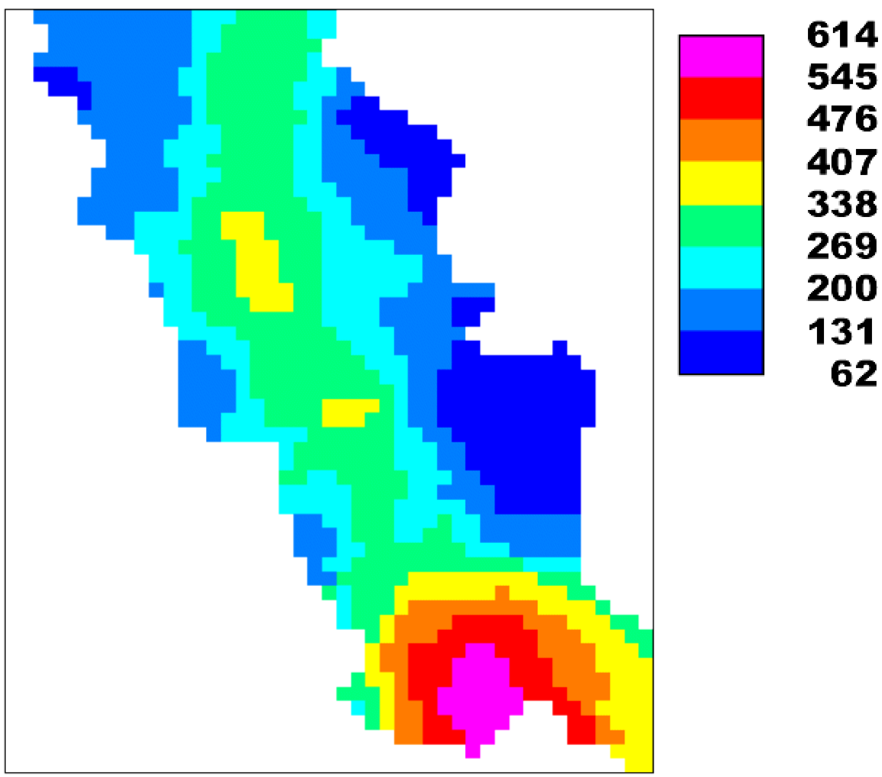
This shows that the output error is a sum of various contributions, each caused by one of the uncertain inputs

This tells you how best to decrease output error

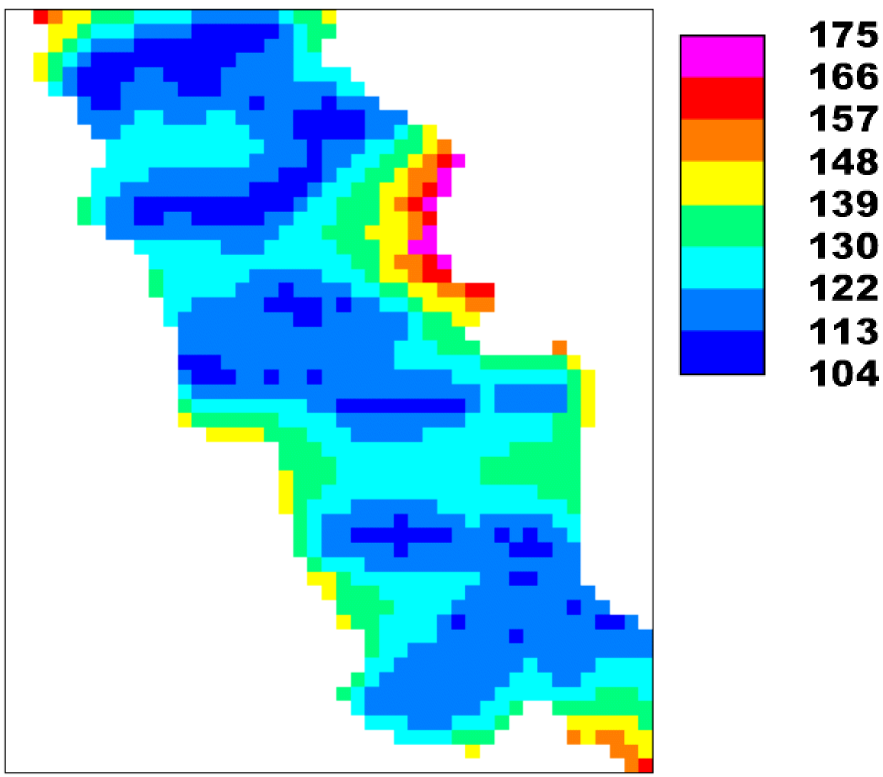
EXERCISE 3

Kriging lead concentration from point observations

prediction (mg/kg)

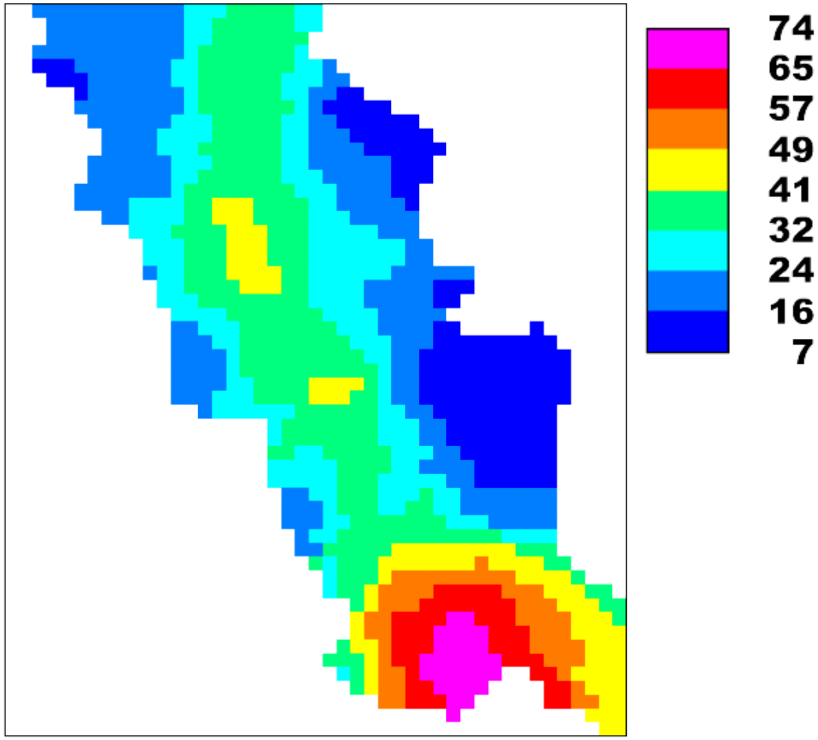


kriging st. dev. (mg/kg)

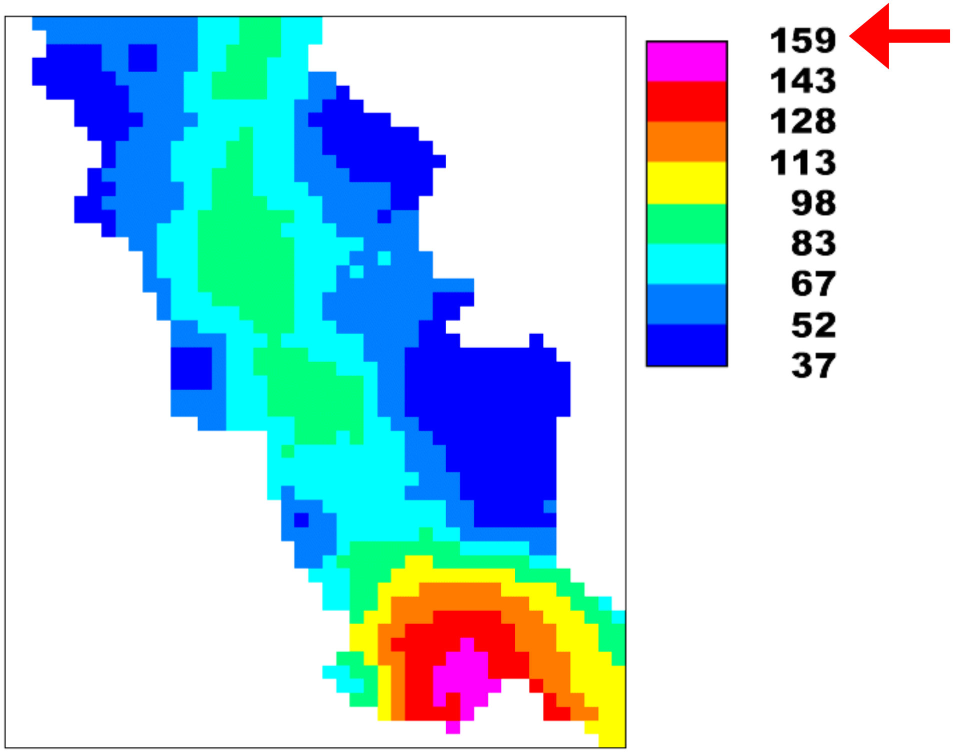


Results of uncertainty analysis for daily lead ingestion

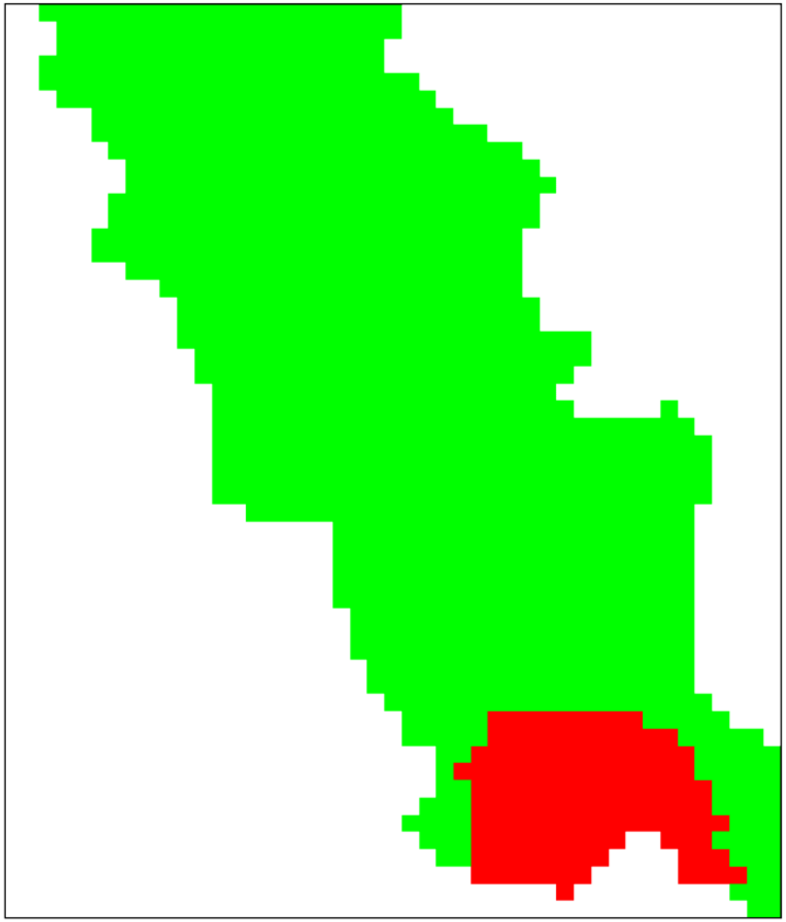
prediction (µg/day)



st. deviation (µg/day)



Red area: expected daily lead ingestion larger than the Acceptable Daily Intake (ADI, 50µg/day)



Red area: P(daily lead ingestion larger than ADI) ≥ 0.05