Residual Error Analysis of GPS Data Sequence Based on WP

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Abstract. GPS observation sequence contains all kinds of impact factors and the function relations between them are complicated. That influences the extraction of feature information and the ability of explanation of parameter models. In residual error that still exists in the signal after a series of professional treatment, including difference correcting, tide correcting, and so on, system error has a relative high value compared to random error. Further more, information of different factors impacting observations behaves different in system error, and shows some periodicity in the frequency domain. If these factors can be departed during positioning and orbit determination, not only will the precision of the two be enhanced but also that can provide materials for the study of other disciplines. Traditional parameter estimation methods in these issues don’t appear so efficiency. In this paper, periodicity of error of GPS data sequence is analyzed, and residual error items with periods of a year, half of a year, a month and half of a month are extracted within different frequency bands obtained by wavelet packet transform. Then the corresponding residual errors are acquired. Frequency aliasing between sub-bands appears during the above-mentioned process. Wavelet filters applied aren’t ideal so that one sub-band obtains some frequencies belong to other near ones and up-sampling and down-sampling these mixed sub-bands will cause frequency folding, for not satisfying the sampling theorem. Both of them lead to the frequency aliasing during decomposition and reconstruction of wavelet packet algorithm. To Eliminate or weaken the impact of aliasing, relevant measures are studied to improve period items quality of GPS data. The availability of method is proved in testing example, and items with period of a year, half of a year, a month and half of a month acquired with it, are more believable.

Keywords: GPS, wavelet packet, frequency aliasing, feature extraction, residual error

1. Introduction

In the modern measurement technology, large-scope, long-time, and even uninterrupted dynamic observation means is provided [9], accuracy of observation is improved continuously, and observation environment is increasingly complex. Contained in dynamic observations, value of system error is bigger, influencing factors of it are larger and function relation of it is more complicated than those of random error. On one hand, the systemic effect can hardly be considered in parameter model, and that affects the interpretation capacity of the model. On the other hand, it is always as a whole removed or compensation as harmful components at present. In fact, system error contains the information of various factors that influence the observation. If we can recognize and extract the information correctly, it can not only help to improve the accuracy of parameter estimation but also provide the basis for the study of other disciplines. In precise positioning and orbit determination of Global Positioning System(GPS), the concept of measurement (phase of satellite signal),includes the distance information from satellites to ground observation points, the time delay happened when electromagnetic signals travel through the ionosphere and troposphere, the influence of signal multipath effects, and so on. If these factors can be departed, that will improve positioning and orbit determination accuracy, and that also provide material for other disciplines’ study, such as Geodynamics, Geophysics. Similar problems in the modern measurement data processing are with the certain universality, and the traditional parameter model appears weak in solving these problems.
In early 1990s, the concept of wavelet is introduced in geodesy, being gained gradually attention, and rich results are achieved with it. It is applied to accurately determine tide gravity parameter, polar shift variation caused by the Earth's rotation and analysis and prediction of ENSO cycle [1]. In the measurement data processing, many scholars using wavelet analysis to detect GPS cycle slip of phase observation[2], extract signal characteristics using the certain correlation between different levels in wavelet decomposition and between signals and the different factors showing some cyclic characteristics that impact the GPS signal propagation[3]-[11]. At present, the use of wavelets analysis method in the periodic signal analysis in deformation data[12], cycle slip of phase detection[13][14], signal de-noising[15]~[20] has improved the precision of data processing. To a certain extent, it has played a positive role in promoting the theory and method of error analysis. Whereas, in the modern geodesy, observing conditions are more and more complex, and precision requirements are increasingly high, signal extraction in background of strong noise, aliasing error identification and separation and other issues have not yet been satisfactorily resolved. For many GPS positioning impact factors being certain cycle and residual error still existing after some processing, we decompose GPS data sequence with the theory of wavelet packets, based on analyzing the residuals cyclical characteristic. During the decomposition, frequency aliasing, such as frequency folding and one sub-band obtained frequencies belonging to the other ones, is studied, and measures are also taken to improve wavelet packet algorithm to eliminate or weaken the impact of aliasing, so that residual factors can be correctly extracted.

2. Analysis of cyclical characteristic of GPS error

2.1. Ionosphere refraction error
Distribution of electron density is uneven in the ionosphere, and it changes along with the difference in altitude, time, season and location of the stations varying. As electronic content changing in a whole day is concerned, it is high during the day (8~18h), and is low at night. For the intensity of solar radiation also changes with the seasonal changing, such as electronic content in ionosphere in July is four times than that in November, so the electronic content has annual change too.

2.2. Troposphere refraction error
Troposphere refractive error is closely related to ground climate, atmosphere pressure, temperature and humidity changes. Temperature change in a day converting to frequency in signal is 1.2e-5HZ. In 24 hours atmosphere changes little, but there is a certain cyclical characteristic. Local atmosphere pressure changes mainly behave as seasonal change.

2.3. Multipath error
Multipath error showed relation to distance and mediums surrounding the receiver antenna, and it also changes with time, showing cyclical features. Period of multipath effects often between several tens of seconds and several tens of minutes (J.M.Tranquilla and J.P.Carr, 1990).

2.4. Satellite ephemeris error
Satellite ephemeris error is defined as the difference between space location given by the satellite ephemeris and actual location. Early, Broadcast ephemeris accuracy is 20~100m, with perturbation force model and orbit determination technology be constantly improving, it is up to 5~10m, or even higher. In one observation period, ephemeris error belong to systematic error, one kind of initial data error, and it is the important source of error at precise relative positioning.

2.5. Relativity effect
According to the general relativity, if the oscillator is put on different equipotential surface, frequency of it will be different because gravitational potential is difference. The phenomenon is called gravitation shift. In addition to satellite clock frequency drift, general relativity includes geometry delay of signal propagation caused by the Earth's gravitational field, usually called gravitation delay. It gains maximum, being about 19mm, When the satellite approaches to the ground level, while it gains minimum, being about 13mm, when satellite nears to stations zenith direction. In a month, as the Moon rotating around the Earth, there are twice when the moon is the most close to the earth, Therefore, the period caused by the moon gravitation is half of
Earth rotation correction correlates to satellite position and the relative position between satellite and receiver, and the error caused by it have strong relativity. However, when the baseline is long enough, the relativity is weaker. This error should be corrected firstly before positioning computation. The period of this impact caused by earth rotation to satellites is a day.

2.7. Earth tide correction
With the role of universal gravitation of the sun and the moon, solid Earth generates cyclical elastic deformation, forming earth tide. In addition, the role also causes the load on the Earth to change cyclically, forming loading tide. In accordance with its affecting period, tide can be divided into daily tide with period of one day, semi-monthly tide with period of 13.7 days, monthly tide with period 27.6 days, semi-yearly tide with period of 182.6 days and yearly tide all caused by the sun.

Polar tide doesn’t exist period of a day, and its impact on station is fundamentally a constant in a day. The greatest change in the year is 10mm, and compared with the ocean tide, very-long-term variable of polar tide is much greater. If the polar tide correction isn’t considered, tide effects will be the main factor that causes the station change seasonally, with period of 15 days.

3. Decomposition and reconstruction theory of wavelet packet
Supposing orthogonal scale function $\phi(t)$ and wavelet function $\psi(t)$ satisfy the following dual scale equations

$$\begin{align*}
\phi(t) &= \sqrt{2} \sum_k h_{0k} \phi(2t - k) \\
\psi(t) &= \sqrt{2} \sum_k h_{1k} \phi(2t - k)
\end{align*}$$

where, $h_{0k} = \langle \phi_{1,k}, \phi(t) \rangle$, $h_{1k} = \langle \phi_{1,k}, \psi(t) \rangle$. $\{h_{0k}\}$ is low-pass filter and $\{h_{1k}\}$ is high-pass filter.

Let $\phi(t) = w_0(t), \psi(t) = w_1(t)$ in the dual scale equations, then

$$\begin{align*}
w_{2n}(t) &= \sqrt{2} \sum_{k \in \mathbb{Z}} h_{0k} w_n(2t - k) \\
w_{2n+1}(t) &= \sqrt{2} \sum_{k \in \mathbb{Z}} h_{1k} w_n(2t - k)
\end{align*}$$

The function set $\{w_n(t)\}_{n \in \mathbb{Z}}$ is called wavelet packet defined by $w_0(t) = \phi(t)$ and its equal function in frequency domain is

$$w_n(\omega) = \prod_{k=1}^{\infty} H'_{n_k}(\omega/2^k),$$

where,

$$\epsilon_k = 0 \text{ or } \epsilon_k = 1$$

$$H_0(\omega) = \frac{1}{\sqrt{2}} H_0(\omega) = \frac{1}{\sqrt{2}} \sum_k h_{0k} e^{-j\omega k}$$

$$H_1(\omega) = \frac{1}{\sqrt{2}} H_1(\omega) = \frac{1}{\sqrt{2}} \sum_k h_{1k} e^{-j\omega k}$$

And such defined wavelet packet satisfies the translation orthogonality $\langle w_n(t - k), w_n(t - l) \rangle = \delta_{kl}$ and orthogonality $\langle w_{2n}(t - k), w_{2n+1}(t - l) \rangle = 0$, $k, l \in \mathbb{Z}$ in the both formulas.

In wavelet multiresolution analysis, Hilbert space $L^2(R)$ is divided as $L^2(R) = \bigoplus_{j \in \mathbb{Z}} W_j$, $W_j$ is the closure spaces of $\psi(t)$. Whereas in wavelet packet analysis, According to (5), and supposing
where, \( m = 0, 1, 2, \ldots, 2^k - 1 \), \( k = 1, 2, 3, \ldots \). The standard orthogonal base of subspace \( U_{j+k}^{2^k} \) is \( \{ 2^{-(j-l)/2} \varphi_{2^j m} (2^{-j} x - k); k \in \mathbb{Z} \} \).

The arithmetic of wavelet packet can be described as: Supposing \( G^j f(t) \in U_n^j \) and \( G^j f(t) = \sum_{l} d_{l, n} \cdot 2^{-j} w_n \left( 2^{-j} - l \right) \), the decomposition recursive formula of wavelet packet is

\[
\begin{align*}
    d_{j+1, 2n}^l &= \sum_{l} h_{n(2j-l)} d_l^n \\
    d_{j+1, 2n+1}^l &= \sum_{l} h_{n(2j-l+1)} d_l^n
\end{align*}
\]

And the reconstruction formula is:

\[
d_{j, n} = \sum_{k} \left[ h_{n(2j-k)} d_{j+1, 2n}^l + h_{n(2j-k)} d_{j+1, 2n+1}^l \right] = \sum_{k} g_0^{l-2k} d_{j+1, 2n}^l + \sum_{k} g_1^{l-2k} d_{j+1, 2n+1}^l
\]

where \( \{ g_0(k), g_1(k) \} = \langle \varphi_{2^j} (t), \psi_{2^j} (t) \rangle \).

**4. GPS observation sequence analysis and residual information extraction**

**4.1. GPS observation sequence analysis**

In order to analyze the frequency aliasing, decompose elevation data of 1995,1996,1998 and 2001 supplied by Shanghai GPS observation station separately to 6 levels with db6 wavelet. Take the first 16 frequency sub-bands to research and they’re showed in Fig.1~Fig.4.In the four figures, The top figure shows the elevation data of 1995,1996,1998,2001 separately and its X-axis unit is day and Y-axis’ is mm; maximum scale is 50, and the minimum is -50;the other figures show the first 16 sub-bands information of the 6th level, and \( L_i \) represents the \( i \)th wavelet packet of the 6th level.

From Fig.1 to Fig.4, every sub-band has its own main frequency and main cycle, although the amplitude is a bit different. Cycle statistics of different sub-band in decomposition of elevation data is showed in Table 1. It can be seen from table 1 that distribution of feature frequencies in the GPS signal is decentralized. That’s because the filters are so nonideal that low frequency sub-band contains a part of frequency components that belong to high frequency ones and high frequency sub-band contains a part of frequency components that belong to low frequency ones, as well. Further more, these redundant frequency components existing in low-frequency sub-band and the high-frequency sub-band will cause frequency folding after down-sampling and up-sampling with 2, because they don’t satisfy the sampling theorem. In order to weaken or even eliminate the aliasing in the wavelet packet algorithm, measures are taken to improve the algorithm.

- **Reordering nodes**

According to sampling theorem, while decompose the detail part (high frequency part) with decomposition filters \( H \) and \( G \) at every level, the high frequency band will be completely folded, calling the phenomenon frequency interleaving. Therefore, not only are there false frequencies in every sub-band, but also the sub-band’ order is in confusion. Fortunately, there is a law of the confusion, that at every level, if decompose the low frequency part, there will not be frequency interleaving, but, there will be, if decompose the high frequency part, and the interleaving at low levels will cause another interleaving at high levels. In accordance with the law, reordering nodes will remove the frequency interleaving.
Table 1. Cycle statistics of different sub-bands in decomposition of elevation data (unit: day)

<table>
<thead>
<tr>
<th>year</th>
<th>L2</th>
<th>L3</th>
<th>L4</th>
<th>L5</th>
<th>L6</th>
<th>L7</th>
<th>L8</th>
<th>L9</th>
<th>L10</th>
<th>L11</th>
<th>L12</th>
<th>L13</th>
<th>L14</th>
<th>L15</th>
<th>L16</th>
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<tbody>
<tr>
<td>1995</td>
<td>103</td>
<td>34</td>
<td>52</td>
<td>16</td>
<td>20</td>
<td>30</td>
<td>26</td>
<td>8</td>
<td>9</td>
<td>11</td>
<td>8</td>
<td>15</td>
<td>14</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>1996</td>
<td>79</td>
<td>35</td>
<td>46</td>
<td>16</td>
<td>20</td>
<td>31</td>
<td>23</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>16</td>
<td>14</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>1998</td>
<td>71</td>
<td>43</td>
<td>53</td>
<td>17</td>
<td>21</td>
<td>30</td>
<td>22</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td>16</td>
<td>14</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>2001</td>
<td>74</td>
<td>34</td>
<td>51</td>
<td>16</td>
<td>17</td>
<td>31</td>
<td>22</td>
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<td>9</td>
<td>16</td>
<td>14</td>
<td>11</td>
<td>13</td>
</tr>
</tbody>
</table>

Fig. 1: the elevation data of 1995 and the first 16 sub-bands information of the 6th level.

Fig. 2: the elevation data of 1996 and the first 16 sub-bands information of the 6th level.

Fig. 3: the elevation data of 1998 and the first 16 sub-bands information of the 6th level.
single sub-band reconstruction algorithm
Reordering nodes can remove the frequency interleaving caused by down-sampling and up-sampling the high frequency not satisfying the sampling theorem, but there are still other frequency folding during decomposition. In order to avoid the left frequency folding, the single sub-band reconstruction algorithm is supplied. It is that decompose the signal with wavelet packet algorithm, and wavelet packet coefficients on every scale can be obtained, then reconstruct wavelet packet coefficients of each sub-band to original signal separately. This algorithm will eliminate the frequency folding by making use of backward folding effect between down-sampling and up-sampling with 2.

improved single sub-band reconstruction algorithm
In single sub-band reconstruction algorithm, there isn’t frequency folding. But the wavelet filters to be used is not ideal, each sub-band gained by wavelet packet transform has some frequencies that should belong to other sub-bands. If false frequencies are got rid of, the reconstruction signal will not exist frequency aliasing after reordering nodes. Based on this, the improved single sub-band reconstruction algorithm is offered, in that use FFT (Fast Fourier Transform) and IFFT (Inverse Fast Fourier Transform) to remove the redundant frequencies in each sub-band.

4.2. Residual information extraction of GPS data sequence
Analyze seven-year GPS elevation data of Shanghai, from 1995 to 2001, with db6 wavelet to extract the residual information with period of a year, half of a year, a month and half of a month, and the residual errors information with different cycles and the corresponding spectrum density extracted by classical wavelet packet transform algorithm and improved wavelet packet transform algorithm are showed separately in Fig 5 and Fig 6. In the both figures, at the left column, X-axis is on behalf of time, with unit of day and Y-axis is on behalf of elevation, with unit of mm. At the right column, X-axis is on behalf of frequency and Y-axis is spectrum density. The first row is the residual error information with annual cycle (left) and its corresponding spectrum density (right); the second row is the residual error information with semi-annual cycle (left) and its corresponding spectrum density (right); the third row is the residual error information with monthly cycle (left) and its corresponding spectrum density (right); the third row is the residual error information with semi-monthly cycle (left) and its corresponding spectrum density (right).

Comparing the residual information and the corresponding spectrum density (Fig. 9, Fig. 10) obtained by the two algorithms, it can be seen clearly that the frequency aliasing in the information extracted by the classical algorithm is serious, for each one has redundant frequencies, while it is greatly reduced in the information extracted by the improved algorithm. The results of the improved algorithm are more believable. This advantage is particularly noticeable in the long-period item. The values of residuals with annual cycle, semi-annual cycle, monthly cycle and semi-monthly cycle are listed in Table 2.
Table 2. Values of residual errors with different cycles based on two algorithms (unit: mm)

<table>
<thead>
<tr>
<th>cycle</th>
<th>classical algorithm</th>
<th>improved algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>annual cycle</td>
<td>6.57</td>
<td>6.49</td>
</tr>
<tr>
<td>semi-Annual cycle</td>
<td>6.81</td>
<td>1.36</td>
</tr>
<tr>
<td>monthly cycle</td>
<td>5.81</td>
<td>2.65</td>
</tr>
<tr>
<td>semi-monthly cycle</td>
<td>7.18</td>
<td>4.29</td>
</tr>
</tbody>
</table>

Fig. 5: the residual errors information with different cycles and the corresponding spectrum density extracted by classical wavelet packet transform algorithm

Fig. 6: the residual errors information with different cycles and the corresponding spectrum density extracted by improved wavelet packet transform algorithm

5. Conclusion

The long-time, uninterrupted dynamic observation data provided by GPS is influenced by a variety of error factors. These factors behave some regularity, and even most of them behaves cyclic feature. By decomposing the GPS data sequence with wavelet packet theory, error items with different period can be projected to the corresponding sub-bands, then the estimate of each error item can be obtained by reconstructing these sub-bands to original signal separately. During the process, the wavelet filter is nonideal,
so that low frequency sub-bands contain high frequencies belonging to high frequency sub-bands, and the high frequency sub-bands contain low frequencies belonging to low frequency sub-bands, and further more the high frequencies contained in the low-frequency sub-bands and the high-frequency sub-bands will cause frequency folding after down-sampling and up-sampling with 2, because they don’t satisfy the sampling theorem. Contraposing these reasons, measures such as reordering nodes, improved single sub-band reconstruction algorithm are taken to eliminate or weaken the frequency aliasing, consequently to improve the quality of estimate of residual in GPS data.

6. References


