Producing digital elevation models with uncertainty estimates using a multi-scale Kalman filter

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Abstract

The Shuttle Radar Topographic Mission (SRTM) digital elevation data provides near-global coverage at about 90 m resolution and in much of the world is now the best available topographic data. Its application for quantitative analysis is limited by random noise and systematic offsets due to vegetation. This paper describes a multiscale Kalman smoothing algorithm for removing vegetation effects and smoothing random variations. The algorithm assimilates dense SRTM data, a vegetation mask and sparser but more accurate ICESat satellite laser altimetry data to produce improved estimates of ground height. The method is found to be effective provided the vegetation mask accurately reflects the location of vegetation-induced offsets in the SRTM data. The method also produces estimates of uncertainty in the elevations, facilitating the use of methods for propagating error through derived terrain attributes.

Keywords: multiscale smoothing, digital elevation model, SRTM, uncertainty estimates

1 Introduction

Terrain data in the form of digital elevation models (DEMs) is fundamental data for understanding and managing natural systems. Appropriate use of terrain data supports understanding of water movement, patterns of soil physical and chemical properties including depth, spatial interpolation of climate, modelling of sediment transport processes, vegetation patterns, and a host of other applications. Most applications of terrain data require information on terrain shape and drainage structure, not just elevation, and measures of shape rely on local differences in elevations (Hutchinson and Gallant, 2000). The impact of elevation errors on these derived shape attributes can be magnified or reduced depending on the spatial correlation of the errors. Random errors cause large errors in derived shape, while systematic errors (offsets) have little or no effect on shape. Small local errors in drainage structure can in some cases result in large changes to the structure of drainage networks or the shapes and areas of catchments. The propagation of errors from DEMs through the derived attributes is frequently ignored, not least because useful estimates of DEM errors and their spatial correlations are not provided with DEM data.

The recent release of the Shuttle Radar Topographic Mission (SRTM) elevation data (NASA, 2006), providing nearly global coverage at 3° resolution (approximately 90 m) provides many new opportunities for applications of DEM data. In Australia, the best continental DEM prior
to the SRTM release was the 9″ DEM (Geoscience Australia, 2005) derived from mapped spot heights, contours and streamlines. The 9″ DEM has been widely used for continental and regional scale analysis and has demonstrated the value of a consistent, high quality national terrain data set.

The SRTM data represents a significant improvement in spatial resolution and detail, particularly in low relief areas, but is not without its own problems. The first is that the radar sensing underlying the SRTM is unable to “see” the ground under dense vegetation and buildings, leading to offsets in heights relative to true ground height. The second is that random variation in heights between adjacent data points corrupt the measurement of important terrain attributes such as slope. These two errors are particularly troublesome when deriving flow directions and stream lines in the flatter agricultural areas of Australia: there is little if any surface slope towards the streams and rivers and they are frequently bordered by trees. As a result, many watercourses appear in the SRTM DEM as higher elevation than the surrounding land.

To obtain maximum value from the SRTM data for use in Australia, the offsets due to trees and buildings need to be removed and some smoothing introduced to improve the reliability of slope and other terrain attributes. Providing estimates of the remaining errors in the elevation data will foster the use of error propagation methods for the derived shape attributes and the applications of that data.

The method chosen to undertake the removal of vegetation offset and adaptive smoothing is the Multiscale Kalman Smoother (Chou, et al., 1994), a data assimilation and optimal estimation algorithm for spatial data.

1.1 Why a multiscale approach?
A multiscale approach for estimating elevations is attractive for several reasons. It allows assimilation of observations at different scales, permitting a coherent integration of data from different sources and with different characteristics. It also provides for smoothing of the observations at scales appropriate to the particular characteristics of the topography and the data.

The ANUDEM algorithm (Hutchinson, 1989, 2000, 2006) for interpolating DEMs from contour, point and streamline data also uses a multi-scale approach in the form of multigrid interpolation, which provides substantial benefits in numerical efficiency and in capturing topographic structures at appropriate scales.

Multiscale approaches to terrain analysis are now being developed (eg Gallant and Dowling, 2003) reflecting the growing realisation that information about the shape of the terrain surface exists at a range of scales, not just at the resolution of the data representing the surface.

2 The Multiscale Kalman Smoother
The Kalman filter (Kalman, 1960) and the Rauch-Tung-Striebel smoother (Rauch et al., 1965) are well established techniques for time-series estimation in linear dynamical systems. These methods provide optimal estimates of the values and covariances of a set of state variables based on a process model and observations. Both the model and the observations have uncertainties and the estimation assimilates the predictions of the model with the observations in a statistically robust manner.
Filtering and smoothing are distinct statistical processes. In a time series sense, filtering is optimal estimation of the current state given current and past observations. Smoothing is optimal estimation of the complete sequence of states given both past and future observations. Clearly filtering can be done in real time because it does not require future information, but smoothing can only be done once the full record is available. The filtering calculations progress forwards in time, while the smoothing calculations progress backwards. The combined process is called Kalman smoothing.

The application of the Kalman smoothing algorithm to a multiscale model is described in Chou et al. (1994). The multiscale model is constructed as a tree with the finest scale – the scale at which estimates are ultimately required – at the leaves of the tree and the coarsest scale – essentially a global aggregation of the entire data set – at the root. For two dimensional data a quadtree is the most obvious implementation, where each non-leaf element contains four finer scale elements, as in Figure 1. Locations within the tree are denoted by an abstract index $s$ and relationships across scale are represented by the operations $\gamma$ and $\alpha$: $\gamma$ is the raising operator (towards coarser scales) so $s\gamma$ is the parent of $s$; $\alpha$ is the lowering operator (towards finer scales) with four versions corresponding to the four children.

![Quadtree structure and node labelling nomenclature.](image)

In this model, the “process” is not concerned with evolution of the state variables through time, but with their evolution across scales. The accepted method is to equate the forwards process direction with a coarsening in scale so a single forward step consists of a prediction of the state variables at the next coarsest scale. The filtering process thus commences at the finest scale and produces an estimate of means and covariances of the state variables at progressively coarser scales. After filtering to the top of the tree, the smoothing process is performed back down the tree to the finest scale.
Note that notation varies between authors. In the following equations, the notation of Fieguth et al., 1999 is used. Lower case letters refer to vectors, upper case to matrices.

The multiscale Kalman smoothing model is based on a downwards process model (from coarse to fine scales):

\[ x(s) = A(s)x(sγ) + B(s)w(s) \]  \hspace{1cm} (1)

where \( s \) is an abstract index for identifying nodes in the tree, \( γ \) is a change-of-scale operator such that \( sγ \) is the parent of \( s \), \( x(s) \) is the vector of state variables, \( A(s) \) is the process update matrix, \( B(s) \) specifies the magnitudes and correlations of the process noise components (so that \( B(s)B^T(s) \) is the noise covariance matrix), and \( w(s) \) is a vector of independent Gaussian normal variates. The \( B(s)w(s) \) term represents the new information introduced by moving from the coarser to finer scale. \( A(s) \) is commonly the identity matrix since the expected values of the state variables often do not change across scale, and \( B(s) \) commonly has non-zero terms only on the diagonal corresponding to uncorrelated components in the new information. Note that if \( A(s) \) is not the identity matrix then the model includes some kind of explicit evolution of \( x \) across scale and the values of \( x \) will change even in the absence of any new information \( B(s)w(s) \). Conversely, if \( A(s) \) is the identity matrix the only changes in \( x \) are those introduced by \( B(s)w(s) \).

Observations are represented as:

\[ y(s) = C(s)x(s) + v(s) \]  \hspace{1cm} (2)

where \( C(s) \) is the observation matrix and \( v(s) \) is the observation noise, assumed to be Gaussian with zero mean and covariance \( R(s) \). Note that the observation vector \( y(s) \) does not have to have the same number of elements as the state vector \( x(s) \): one of the advantages of Kalman smoothing and the MKS data assimilation scheme is that it can estimate the values of state variables that are not directly observable. Also note that observations can be irregularly spaced and are only incorporated where they are available.

It is also important to note that \( A \) and \( B \) can vary both in scale and location, and \( C \) and \( R \) can be different for every observation. This provides a great deal of flexibility in specifying the model and permits a variety of different observations to be incorporated.

The forward process of multiscale Kalman filtering proceeds from the fine to coarse scales, in the reverse direction to (1), so an upwards (fine-to-coarse) model must be constructed from the downwards model of (1):

\[ x(sγ) = F(s)x(s) + π(s) \]  \hspace{1cm} (3)

\[ Q(s) = E[π(s)π^T(s)] \]  \hspace{1cm} (4)

\( F(s) \) and \( Q(s) \) are calculated from \( A(s) \), \( B(s) \) and the prior covariance at the coarsest scale, \( P_0 \).
The details of the estimation scheme will not be presented here; full details are presented in Chou et al. (1994), Fieguth et al. (1999) and Kumar (1999).

The method has been applied in a variety of contexts including TOPEX/POSEIDON sea level data (Fieguth et al., 1995); soil moisture data (Kumar, 1999); and terrain data from radar and laser altimeters (Slatton et al., 2001).

2.1 The MKS model for assimilating SRTM, ICESat and vegetation data

The objective of the work described here is to improve the utility of the 3″ SRTM DEM by removing the effects of vegetation and systematic offsets and reducing the random noise.

The model for SRTM observations is:

\[ z_R(x,y) = f(x,y) + m(x,y)g(x,y) + h(x,y) + \epsilon_R \] (5)

where \( z_R \) is the elevation reported in the SRTM DEM, \( f \) is the actual ground elevation at that location, \( g \) is the offset introduced by vegetation, \( m \) is a mask value between 0 and 1 indicating the influence of vegetation, \( h \) is the systematic height offset (including vertical datum differences) and \( \epsilon_R \) is the observation error, assumed to be normally distributed with a standard deviation of \( \sigma_R \). The vegetation mask is provided as a data layer at the same resolution as the SRTM DEM and is assumed to be correct: it is not subject to estimation by the MKS algorithm. The vegetation height \( g \) and systematic offset \( h \) vary only slowly through space compared to \( f \) and \( m \).

The state vector is:

\[ x = [f \ g \ h] \] (6)

For the SRTM data, the observation vector is:

\[ C = [1 \ m \ 1] \] (7)

The process evolution matrix \( A \) is the 3 x 3 identity matrix. \( B(s) \) can be determined from an analysis of the data (see Model Parameters, below).

The SRTM and vegetation observations are supplemented by sparse but more accurate ICESat (ref) observations of ground elevation and vegetation height:

\[ z_L(x,y) = f(x,y) + \epsilon_L \] (8)

\[ z_V(x,y) = m(x,y)g(x,y) + \epsilon_V \] (9)

Note that the laser altimeter instrument (GLAS) on the ICESat platform is, in most cases, able to penetrate foliage cover to obtain a ground return and that the processed signal includes an estimation of vegetation height separately from terrain elevation. This data source allows estimation of the slowly varying offset term \( h \) and helps to define \( g \).

A more elaborate model has also been developed to permit direct estimation of the first derivatives of surface height, \( f_x \) and \( f_y \). The intention is that this model will allow direct
estimation of the first derivatives, and hence of slope and slope direction (aspect). The observation models are:

\[
\begin{align*}
    z_x(x,y) &= f(x_0,y_0) + m(x,y)g(x,y) + \delta h(x,y) + (x-x_0)f_x + (y-y_0)f_y + \epsilon_x \\
    z_y(x,y) &= f(x_0,y_0) + (x-x_0)f_x + (y-y_0)f_y + \epsilon_y
\end{align*}
\]

where \((x_0,y_0)\) is the location of the grid point nearest the observation point and \((x,y)\) is the location of the observation itself. Location offsets in \(g\) and \(h\) are ignored because they are assumed to be more spatially coherent than \(f\). Under this model the evolution matrix \(A\) in is no longer the identity matrix and incorporates the contributions of \(f_x\) and \(f_y\) due to the change in location of the centre of the cell as part of the change of scale.

An important effect of including \(f_x\) and \(f_y\) is that the finer scale variations in height can be accounted for by these derivative terms and not by random variations. This helps to smooth the estimated \(f\) surface, and the degree of smoothing at different scales can be controlled by the specification of \(B\).

### 2.2 Model parameters

The MKS algorithm has been tested using simulated SRTM data generated according to the model of (3). An existing 10 m resolution DEM in a low relief landscape was aggregated to 90 m resolution to provide true heights \(f\). A woody vegetation classification, derived from remote sensed imagery at 30 m resolution, was aggregated to 90 m resolution to form \(m\). A constant vegetation height of 10 m was used for \(g\) and a constant offset of 5 was used for \(h\). Random Gaussian noise with a standard deviation of 1 m was then added. For testing, a small area of 64 × 64 cells was generated. A single higher precision ICESat-style data point is included as a separate observation to allow estimation of the constant offset. It is located in a vegetation-free area so it does not contribute to the estimation of vegetation height.

Effective use of the MKS algorithm depends on proper parameterisation of the model. The \(A\) (evolution) and \(C\) (observation) matrices are dictated by the structure of the model, and the \(R\) (observation covariance) matrix is based on knowledge of the measurement uncertainties. The specification of the SRTM data states that the standard deviation is less than 16 m but in the area examined in this paper the uncertainty (apart from vegetation effects) appears to be much lower than this so the variance for simulated SRTM observations is set to 4 m². The ICESat data is more precise and the variances for ground elevation and vegetation height for these simulations are set to 0.2 m² and 1 m² respectively. Off-diagonal elements of \(R\) are set to 0.

The \(B\) matrix, which specifies the magnitude of additional information that appears in the coarse-to-fine refinement step of (1), is probably the most difficult to estimate but has a substantial influence on the quality of the results. Fieguth et al. (1995), Kumar (1999) and Slatton et al. (2001) all assume an underlying fractal model that implies a power law relationship between \(B\) and scale, although Slatton et al. (2001) also explored direct estimation of multiscale noise. The latter approach is taken here, using a multiscale nested variance analysis of the source data (\(f\)). For each element in the quadtree, the variance is
calculated as a combination of the variance within each of the child elements and the variance of the mean values between the children. At the finest scale, zero or an *a priori* estimate of variance can be used. Using this approach, a between-group and within-group variance is obtained for each cell in each level of the quadtree. The mean value of the between-group variance, corresponding to the new information added at that level of the tree, is converted to a standard deviation for use in $B$.

The method is applied to $f$ for model (5) and to $f_x$ and $f_y$ for model (10). The standard deviations for $f$ in model (10) are calculated from the differences between mean $f$ at each scale and the value estimated from $f$ at the next coarser scale modified by the coarser scale $x_f$ and $y_f$. Table 1 shows the diagonal values of $B$ for each scale in the two models.

<table>
<thead>
<tr>
<th>Scale (m)</th>
<th>Model (5)</th>
<th>Model (10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$</td>
<td>$g$</td>
</tr>
<tr>
<td>2880</td>
<td>3.54</td>
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<tr>
<td>1440</td>
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</tr>
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</tr>
<tr>
<td>90</td>
<td>0.62</td>
<td>0.12</td>
</tr>
</tbody>
</table>

### 3 Results

Figure 2 shows the original surface, the vegetation mask, the simulated observations, the estimated surface and estimated standard deviation. The estimated vegetation height and offset (and standard deviations) are 17.8 (0.85) m and 4.3 (1.0) m, compared to the actual values of 20 m and 5 m – they vary little across space. The key results are that the MKS has removed most of the effects of the vegetation offset and has significantly smoothed the surface compared to the observations. Some blockiness is evident in the surface resulting from the quadtree structure. The standard deviation surface shows a slightly higher standard deviation where vegetation is present and lower standard deviation where the single point of higher accuracy data is present.

Figure 3 shows the dramatic improvement in slope, a key terrain shape parameter, by the smoothing algorithm. While the slope derived from the estimated elevations is still significantly affected by random noise in the elevations, some of the more significant features have been recovered.

Figure 4 shows the effect of incorporating $f_x$ and $f_y$ into the model. The elevation surface is smoother but somewhat blockier, but a further significant reduction in noise in the estimated slope has been achieved and some of the subtler features apparent in Figure 3(a) reappear in Figure 4(c).
Figure 2 MKS smoothing of simulated data ($64 \times 64$ 90 m cells) using the simple model (5). (a) Original surface (lighter shades are higher), (b) Vegetation mask (darker shades are higher proportion of vegetation), (c) Elevation surface corrupted by vegetation (20 m height), offset (5 m) and noise with 2 m standard deviation, and (d) Elevation surface estimated by MKS. (e) Estimated standard deviation of elevation.

Figure 3 Slope from (a) original elevations, (b) elevations corrupted by vegetation and noise, and (c) elevations estimated by the MKS smoother.

Figure 4 Estimated elevation and slope using the more elaborate model (10). (a) Original elevations, (b) estimated elevations, and (c) slope from estimated elevations.
4 Discussion

Direct estimation of slope from the estimated first derivatives of elevation, $f_x$ and $f_y$ (not shown here) is noisier than slopes derived from estimated elevations. This is contrary to our expectation that inclusion of first derivatives in the smoothing process would result in more reliable estimation of those derivatives than is normally achieved using finite difference calculations on noisy data. This aspect of the MKS performance may be achieved with better parameterisation of the new information matrix $B$, but at this stage it is not clear that the additional complexity of this model is warranted. Increasing the number of state variables from 3 to 5 increases the number of matrix elements in $A$, $P$, $F$ and $Q$ from 9 to 25, substantially increasing memory and computational requirements.

Experiments not shown here have demonstrated that, even without ICESat observations, identification of the vegetation height $g$ is possible provided that there are sufficient observations of both unvegetated and vegetated ($m \approx 0$ and $m \approx 1$) locations, and as long as the vegetation mask $m$ is accurate. This separation is facilitated by the difference in scales of variation between elevation $f$ and vegetation height $g$: the vegetation height is assumed to be more spatially coherent than elevation, so the $B$ values for $g$ are substantially smaller than for $f$. Separation of ground elevation $f$ and the systematic offset $h$ without additional observations is not possible, but a single observation of $f$ alone via equation (8) is sufficient to identify $h$.

Other experiments with imperfect vegetation data show that separation of the vegetation height from terrain height is critically dependent on the accuracy of the vegetation mask. A global shift of the vegetation mask by a single pixel is sufficient to prevent the successful separation of vegetation and terrain heights, resulting in an estimated vegetation height close to 0.

5 Conclusions

The Multiscale Kalman Smoothing algorithm is demonstrated to be an effective tool for separating the offset due to vegetation from true ground elevations under the assumptions of models (5) and (10). A significant challenge is the need for accurate information on the location of these vegetation effects. It is unlikely that optically remotely sensed vegetation mapping on its own will match the SRTM observations sufficiently accurately. One approach would be to supplement the optical remote sensing with information derived from the SRTM data itself, separately from the MKS process, to obtain a better indication of where vegetation is affecting the SRTM heights. Another is to allow uncertainty in the vegetation mask and include it in the estimation process, but this introduces non-linear behaviour into the model that requires more complex methods such as the extended Kalman filter.

Obtaining optimum performance from the MKS approach will probably require methods for estimating $B$ and $R$, the scaling information and observation errors. We expect that observation variances $R$ depend to some extent on relief and that the scaling parameters in $B$ depend on geomorphic properties that vary over space, such as hillslope length.

Production of a high quality DEM based on the SRTM 3" data will probably require a combination of techniques. We envisage using the MKS method to deal with the vegetation height offsets and reduce the noise, followed by further smoothing and drainage enforcement.
using ANUDEM. The ANUDEM algorithm may be modified to utilise the spatially varying uncertainty estimates from the MKS smoother.

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References


